I am sure everyone has had truly remarkable experiences that range from the sublime to the grotesque. These experiences may have occurred at home, in the work place, and even while on vacation. In my case when I developed the practical design procedure for an operational amplifier summer it was a significant life-changing experience. Like most I really didn't know that I was even having "an experience" until I communicated that experience to someone who was seriously more knowledgeable about operational amplifiers than I. That someone was my professor, Dr. Gene H. Hostetter. He encouraged me to create a report to document the design procedure I had discovered. Dr. Hostetter told me that in the twenty years he had been teaching this subject he had not seen such a design procedure. That original report, dated September 12, 1980, is shown below.

While I was working on my Master's Degree in Electrical Engineering one of my professors, Dr. Clem J. Savant, Jr., encouraged me to submit my operational amplifier report to the IEEE Region 6 Student Paper contest held at California State University, Northridge, on September 16, 1982. He and my parents accompanied me to the University campus and watched me present my report to the Region 6 judges. My participation in this contest is shown after the September 12, 1980, report. Incidentally, I placed first and won an HP programmable calculator. I attended WESCON that year at the Anaheim Convention Center in Anaheim, California, with Dr. Savant and presented my report to the WESCON judges. I placed third nationally. Sometime later Dr. Savant began preparing his manuscript for his next Electrical Engineering book and asked me for permission to include my operational amplifier report in that book. The relevant pages in that book that contain my report are shown after my Region 6 participation award.

THE OPERATIONAL AMPLIFIER SUMMER
A PRACTICAL DESIGN PROCEDURE

W. Philip Vrbancic, Jr.

Student: Department of Electrical Engineering
California State University
Long Beach

September 12, 1980

THE OPERATIONAL AMPLIFIER SUMMER
A PRACTICAL DESIGN PROCEDURE

Presented here is a practical design procedure for calculating the values of the required resistors for a multiple input operational amplifier summer. Whether the design calls for a nominal valued resistance to be used for an input or for a particular resistance to be seen by the input terminals of the amplifier can easily be realized, and usually by inspection.

Figure 1. shows a generalized form of the operational amplifier summer.

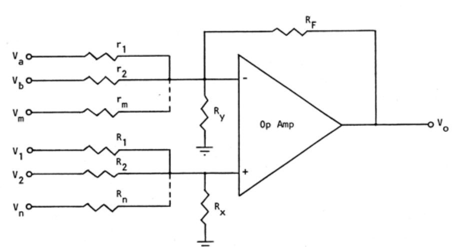


Figure 1.

The desired output voltage, V_o, can be expressed as follows:

$$V_o = X_1V_1 + X_2V_2 +++ X_nV_n - Y_1V_a - Y_2V_b --- Y_mV_m \qquad \text{Eq. 1}$$

where X_1 through X_n and Y_1 through Y_m are the desired gains of the respective input voltages. Let us now define several variables.

$$X = \sum_{i=1}^{n} X_i \qquad \text{Eq. 2}$$

$$Y = \sum_{j=1}^{m} Y_j \qquad \text{Eq. 3}$$

$$Z = X - Y - 1 \qquad \text{Eq. 4}$$

where X_i is a gain to the noninverting input and Y_j is a gain to the inverting input of the operational amplifier.

The value of the resistors connected to the noninverting input as well as those connected to the inverting input are inversely proportional to the required gain of the respective input voltages. That is, a particular input requiring substantial gain as opposed to one that requires little gain would require a smaller input resistance. From operational amplifier theory when the gain of the operational amplifier, G, is very large the output voltage may be written in terms of the resistors connected to the operational amplifier:

$$V_o = \left(1 + \frac{R_F}{R_A}\right)(R_1 \| R_2 \| \cdots \| R_n \| R_x)\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} +++ \frac{V_n}{R_n}\right) - R_F\left(\frac{V_a}{r_1} + \frac{V_b}{r_2} +++ \frac{V_m}{r_m}\right) \qquad \text{Eq. 5}$$

where $R_A = r_1 \| r_2 \| \cdots \| r_m \| R_y$. Eq. 6

Let $K = \left(1 + \frac{R_F}{R_A}\right)(R_1 \| R_2 \| \cdots \| R_n \| R_x)$. Eq. 7

R_F is the proportionality constant for the inverting inputs and K is the proportionality constant for the noninverting inputs for relating a required gain to its input resistor.

To minimize dc bias current offset the noninverting and inverting input terminals must "see" the same input resistance. This constraint, in general form, may be expressed as follows:

$$R_1 \| R_2 \| \cdots \| R_n \| R_x = r_1 \| r_2 \| \cdots \| r_m \| R_y \| R_F. \qquad \text{Eq. 8}$$

Substituting in Eqs. 6 and 7,

$$\frac{K}{1 + \frac{R_F}{R_A}} = R_A \| R_F \qquad \text{Eq. 9}$$

Rearranging,

$$\frac{R_A K}{R_A + R_F} = \frac{R_A R_F}{R_A + R_F} \qquad \text{Eq. 10}$$

$$K = R_F \qquad \text{Eq. 11}$$

Since Eqs. 1 and 5 are both equal to V_o, it follows that

$$R_i = \frac{K}{X_i} = \frac{R_F}{X_i} \qquad \text{Eq. 12}$$

$$r_j = \frac{R_F}{Y_j} \qquad \text{Eq. 13}$$

Returning to Eq. 8, it may be rewritten as follows:

$$\frac{1}{\frac{1}{R_x} + \sum_{i=1}^{n} \frac{1}{R_i}} = \frac{1}{\frac{1}{R_F} + \frac{1}{R_y} + \sum_{j=1}^{m} \frac{1}{r_j}} \qquad \text{Eq. 14}$$

or $\frac{1}{R_x} + \sum_{i=1}^{n} \frac{1}{R_i} = \frac{1}{R_F} + \frac{1}{R_y} + \sum_{j=1}^{m} \frac{1}{r_j}$. Eq. 15

Substituting in Eqs. 12 and 13,

$$\frac{1}{R_x} + \sum_{i=1}^{n} \frac{X_i}{R_F} = \frac{1}{R_F} + \frac{1}{R_y} + \sum_{j=1}^{m} \frac{Y_j}{R_F} \qquad \text{Eq. 16}$$

$$\frac{1}{R_x} + \frac{1}{R_F}\sum_{i=1}^{n} X_i = \frac{1}{R_F} + \frac{1}{R_y} + \frac{1}{R_F}\sum_{j=1}^{m} Y_j . \qquad \text{Eq. 17}$$

Substituting in Eqs. 2 and 3,

$$\frac{1}{R_x} + \frac{1}{R_F}X = \frac{1}{R_F} + \frac{1}{R_y} + \frac{1}{R_F}Y . \qquad \text{Eq. 18}$$

Depending upon the presence of R_x or R_y there are three design possibilities: when $R_x = \infty$, when $R_y = \infty$, when $R_x = R_y = \infty$.

<u>Case 1</u>: $R_x = \infty$, Eq. 18 now becomes:

$$\frac{1}{R_F}X = \frac{1}{R_F} + \frac{1}{R_y} + \frac{1}{R_F}Y . \qquad \text{Eq. 19}$$

Rearranging,

$$\frac{1}{R_y} = \frac{1}{R_F}X - \frac{1}{R_F}Y - \frac{1}{R_F} , \qquad \text{Eq. 20}$$

$$\frac{1}{R_y} = \frac{1}{R_F}(X - Y - 1), \qquad \text{Eq. 21}$$

$$\frac{1}{R_y} = \frac{1}{R_F}(Z), \qquad \text{Eq. 22}$$

$$R_y = \frac{R_F}{Z} , \; Z > 0. \qquad \text{Eq. 23}$$

These results may now be summarized: when $Z > 0$ then

$$R_x = \infty , \; R_y = \frac{R_F}{Z} , \; R_i = \frac{R_F}{X_i} , \; r_j = \frac{R_F}{Y_j} . \qquad \text{Eq. 24}$$

<u>Case 2</u>: $R_y = \infty$, Eq. 18 now becomes:

$$\frac{1}{R_F} + \frac{1}{R_F}X = \frac{1}{R_F} + \frac{1}{R_F}Y . \qquad \text{Eq. 25}$$

Rearranging,

$$\frac{1}{R_x} = -\frac{1}{R_F}X + \frac{1}{R_F}Y + \frac{1}{R_F} , \qquad \text{Eq. 26}$$

$$\frac{1}{R_x} = -\frac{1}{R_F}(X - Y - 1), \qquad \text{Eq. 27}$$

$$\frac{1}{R_x} = -\frac{1}{R_F}(Z), \qquad \text{Eq. 28}$$

$$R_x = \frac{R_F}{-Z} , \; Z < 0. \qquad \text{Eq. 29}$$

These results may now be summarized: when $Z < 0$ then

$$R_y = \infty , \; R_x = \frac{R_F}{-Z} , \; R_i = \frac{R_F}{X_i} , \; r_j = \frac{R_F}{Y_j} .$$

<u>Case 3</u>: $R_x = R_y = \infty$, Eq. 18 now becomes:

$$\frac{1}{R_F}X = \frac{1}{R_F} + \frac{1}{R_F}Y . \qquad \text{Eq. 30}$$

Rearranging,

$$0 = \frac{1}{R_F}X - \frac{1}{R_F}Y - \frac{1}{R_F} , \qquad \text{Eq. 31}$$

$$0 = \frac{1}{R_F}(X - Y - 1), \quad \text{Eq. 32}$$

$$0 = \frac{1}{R_F}(Z), \quad \text{Eq. 33}$$

$$Z = 0 \quad \text{Eq. 34}$$

These results may now be summarized: when Z = 0 then

$$R_x = R_y = \infty, \; R_i = \frac{R_F}{X_i}, \; r_j = \frac{R_F}{Y_j}.$$

The results of all three cases may be combined as follows:

When $Z = X - Y - 1$

$R_y = \frac{R_F}{Z}$ iff $Z>0$, not used otherwise

$R_x = \frac{R_F}{-Z}$ iff $Z<0$, not used otherwise

$R_i = \frac{R_F}{X_i}$

$r_j = \frac{R_F}{Y_j}$

Why are these results so simple, and furthermore, what makes them work? Assume for a moment that $Z>0$ requiring resistor R_y. R_y and R_F may be viewed as also being input signal sources for the inverting terminal having voltage gains of $\frac{R_F}{R_y}$ and $\frac{R_F}{R_F}$ respectively. The operational amplifier will appear balanced when the sum of the desired gains of the signals appearing at both the noninverting and the inverting terminals are equal, and when each terminal sees an equal resistance. Bear in mind, however, that each input signal is

equally amplified -- it is the proportional amount of signal reaching a particular terminal that determines its final gain. With $Z>0$ then

$$X = Y + \frac{R_F}{R_y} + \frac{R_F}{R_F}. \quad \text{Eq. 35}$$

Rearranging,

$$\frac{R_F}{R_F} = X - Y - 1. \quad \text{Eq. 36}$$

Substituting in Eq. 4,

$$\frac{R_F}{R_y} = Z \quad \text{Eq. 37}$$

Therefore $R_y = \frac{R_F}{Z}$ which is identical to Eq. 23, the result found in Case 1. The results of Cases 2 and 3 may be verified in a similar manner.

To complete the actual circuit design a suitable value for R_F must be determined. Once R_F is determined all other resistor values are fixed. Three possible design approaches may be used.

<u>Design Approach 1</u>:

Let R_F = any number, $\neq 0$.

<u>Design Approach 2</u>:

It is desired that no resistor connected to either input terminal of an operational amplifier have a resistance less than a specified value, R. Let K_{max} equal the greatest gain desired of any input, X_i, Y_j, or Z.

Then $R_F = R(K_{max})$.

<u>Design Approach 3</u>:

It is desired that neither input terminal of an operational amplifier see a total input resistance no less than a specified value, R. Let K_{max} equal X or (Y + 1), whichever is greater.

Then $R_F = R(K_{max})$.

The following examples demonstrate the ease at which an operational amplifier summer may be designed.

<u>Example 1</u>: $V_o = 10V_1 + 6V_2 + 4V_3 - 5V_a - 2V_b$. Use Design Approach 2 with R = 10KΩ.

$X = 10 + 6 + 4 = 20$

$Y = 5 + 2 = 7$

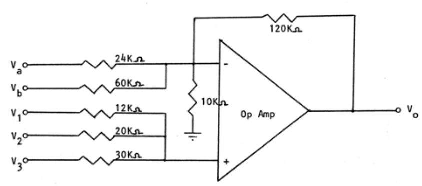
$Z = 20 - 7 - 1 = 12$ ($Z>0$, use R_y)

$K_{max} = 12$

$R_F = 10K\Omega(12) = 120K\Omega$

$R_1 = \frac{120K\Omega}{10} = 12K\Omega$, $R_2 = \frac{120K\Omega}{6} = 20K\Omega$, $R_3 = \frac{120K\Omega}{4} = 30K\Omega$,

$r_1 = \frac{120K\Omega}{5} = 24K\Omega$, $r_2 = \frac{120K\Omega}{2} = 60K\Omega$, $R_y = \frac{120K\Omega}{12} = 10K\Omega$.



<u>Example 2</u>: $V_o = 4V_1 + V_2 - 8V_a - 6V_b$. Use Design Approach 3 with R = 10KΩ.

$X = 4 + 1 = 5$

$Y = 8 + 6 = 14$

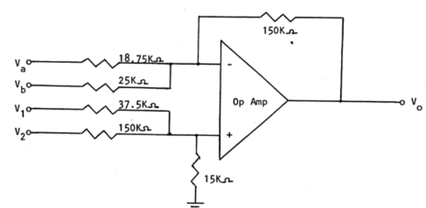
$Z = 5 - 14 - 1 = -10$ ($Z<0$, use R_x)

$K_{max} = (14 + 1) = 15$

$R_F = 10K\Omega(15) = 150K\Omega$

$R_1 = \frac{150K\Omega}{4} = 37.5K\Omega$, $R_2 = \frac{150K\Omega}{1} = 150K\Omega$, $r_1 = \frac{150K\Omega}{8} = 18.75K\Omega$,

$r_2 = \frac{150K\Omega}{6} = 25K\Omega$, $R_x = \frac{150K\Omega}{10} = 15K\Omega$.



THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, INC.
REGION 6

November 5, 1982

Please Reply To:
Forrest L. Staffanson
Electrical Engineering Dept
MEB 3053
University of Utah
Salt Lake City, UT 84112
(801) 581-6941

Philip Vrbancic
4732 Eastbrook Avenue
Lakewood, CA 90713

Dear Mr. Vrbancic:

Enclosed is your certificate for participation in the IEEE Region 6 Student Paper contest at WESCON on September 16, 1982. Congratulations on your demonstrated proficiency and your promise for the profession.

We extend our best wishes for success in the future, and look forward to your continued participation and leadership in IEEE.

Sincerely,

FORREST L. STAFFANSON
Chairman, Student Activities
Region 6, IEEE

FLS:mkl

Enclosure

BUSINESS OFFICE: IEEE Region 6 Office, 701 Welch Road, Suite 2205, Palo Alto, CA 94304
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W. Philip Vrbancic, Jr.

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Participant
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Student Paper Competition for presentation
of his paper entitled
The Operational Amplifier Summer
A Practical Design Procedure

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ELECTRONIC CIRCUIT DESIGN

An Engineering Approach

C. J. Savant, Jr.
California State University, Los Angeles

Martin S. Roden
California State University, Los Angeles

Gordon L. Carpenter
California State University, Long Beach

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On a project of this complexity, it is no simple task to create a finished book out of a manuscript. However, in the able hands of George and Wendy Calmenson of San Francisco, this critical task did indeed look simple. Their professionalism and attention to detail have contributed to a book of which we are proud.

We truly hope that each of the people who contributed to this book and had a hand in its development are as satisfied with the finished product as we are.

Gordon L. Carpenter
Martin S. Roden
C. J. Savant, Jr.

8.6 Design of Op-Amp Circuits

Given the configuration of an op-amp system, we *analyze* that system to determine the output in terms of the inputs using the procedure of Section 8.1.1.

If you wish to *design* a circuit that combines both inverting and noninverting inputs, the problem is more complex. We present a practical design technique in this section.* This technique allows us to design an op-amp summing circuit without elaborate solution of simultaneous equations.

In a design problem, a desired linear equation is given, and the op-amp circuit must be designed. The desired output of the op-amp summer can be expressed as a linear combination of inputs,

$$v_o = X_1v_1 + \cdots + X_nv_n - Y_av_a - \cdots - Y_mv_m \tag{8.8}$$

where $X_1, X_2, \ldots, X_n$ are the desired gains at the noninverting inputs and $Y_a, Y_b, \ldots, Y_m$ are the desired gains at the inverting inputs.

Equation (8.8) is easily implemented with the circuit of Figure 8.10. Equation (8.8) shows that the values of the resistors $R_a, R_b, \ldots, R_m$ and $R_1, R_2, \ldots, R_n$ are inversely proportional to the desired gains associated with the respective input voltages. In other words, if a large gain is desired at a particular input terminal, then the resistance at that terminal is small.

When the open-loop gain of the operational amplifier, G, is large, the output voltage may be written in terms of the resistors connected to the operational amplifier (see equation (8.7)).

$$v_o = \left[1 + \frac{R_F}{R_A}\right](R_1 \| R_2 \| \cdots \| R_n \| R_x)\left[\frac{v_1}{R_1} + \frac{v_2}{R_2} + \cdots + \frac{v_n}{R_n}\right] - R_F\left[\frac{v_a}{R_a} + \frac{v_b}{R_b} + \cdots + \frac{v_m}{R_m}\right] \tag{8.9}$$

where

$$R_A = R_a \| R_b \| \cdots \| R_m \| R_y \tag{8.10}$$

Let us define

$$R_{eq} = \left(1 + \frac{R_F}{R_A}\right)(R_1 \| R_2 \| \cdots \| R_n \| R_x) \tag{8.11}$$

* This technique was devised by Phil Vrbancic, a student at California State University, Long Beach, and was presented in a paper submitted to the *IEEE Region VI 1982 Prize Paper Contest.*

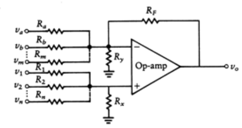

Figure 8.10
Multiple-input summer.

We see that the output voltage is a linear combination of inputs, where each input is divided by its associated resistance and multiplied by another resistance. The multiplying resistance is R_F for inverting inputs and R_{eq} for noninverting inputs.

The Thevenin resistance looking back from the inverting input is usually made equal to that looking back from the noninverting input. We find in Section 9.1 that this constraint minimizes the *dc bias current offset*. For the configuration shown in Figure 8.10, this constraint may be expressed as follows:

$$R_1 \| R_2 \| \cdots \| R_n \| R_x = R_a \| R_b \| \cdots \| R_m \| R_y \| R_F \tag{8.12}$$

Substituting equations (8.10) and (8.11) into equation (8.12) yields

$$\frac{R_{eq}}{1 + R_F/R_A} = R_A \| R_F \tag{8.13}$$

from which we obtain,

$$R_{eq} = R_F \tag{8.14}$$

By comparing like terms in equations (8.8) and (8.9), we obtain the noninverting and inverting gains as follows:

$$X_i = \frac{R_{eq}}{R_i} = \frac{R_F}{R_i} \tag{8.15}$$

and

$$Y_j = \frac{R_F}{R_j} \tag{8.16}$$

The bias-offset relationship, equation (8.12), may be rewritten as follows:

$$\frac{1}{1/R_x + \sum_{i=1}^{n} 1/R_i} = \frac{1}{1/R_F + 1/R_y + \sum_{j=a}^{m} 1/R_j} \tag{8.17}$$

or

$$\frac{1}{R_x} + \sum_{i=1}^{n}\frac{1}{R_i} = \frac{1}{R_F} + \frac{1}{R_y} + \sum_{j=a}^{m}\frac{1}{R_j} \tag{8.18}$$

Substituting equations (8.15) and (8.16) into equation (8.18), we obtain

$$\frac{1}{R_x} + \sum_{i=1}^{n}\frac{X_i}{R_F} = \frac{1}{R_F} + \frac{1}{R_y} + \sum_{j=a}^{m}\frac{Y_j}{R_F}$$

$$\frac{1}{R_x} + \frac{1}{R_F}\sum_{i=1}^{n}X_i = \frac{1}{R_F} + \frac{1}{R_y} + \frac{1}{R_F}\sum_{j=a}^{m}Y_j \tag{8.19}$$

Recall that our goal is to solve for the Rs in terms of the X_i and Y_j. Let us define

$$X = \sum_{i=1}^{n} X_i \tag{8.20}$$

and

$$Y = \sum_{j=a}^{m} Y_j$$

We can then rewrite equation (8.19) as follows:

$$\frac{1}{R_x} + \frac{1}{R_F}X = \frac{1}{R_F} + \frac{1}{R_y} + \frac{1}{R_F}Y \tag{8.21}$$

This is a starting point for our step-by-step design procedure. Recall that R_x and R_y are the resistors between ground and the noninverting and inverting inputs, respectively. The feedback resistor is denoted by R_F.

We can eliminate either or both of the resistors R_x and R_y from the circuit of Figure 8.10. That is, either or both of these resistors can be set to infinity (i.e., open-circuited). This yields three design possibilities, which we denote by Case I, Case II, and Case III. Depending upon the desired multiplying factors relating output to input, one of these cases will yield the appropriate design. The following results are summarized in Table 8.1.

Case I If $R_x \to \infty$, equation (8.21) becomes

$$\frac{X}{R_F} = \frac{1}{R_F} + \frac{1}{R_y} + \frac{Y}{R_F} \tag{8.22}$$

When we choose an R_F, R_y is the only unknown in this equation. Solving for this, we obtain

$$\frac{1}{R_y} = \frac{X}{R_F} - \frac{Y}{R_F} - \frac{1}{R_F} = \frac{(X - Y - 1)}{R_F} \tag{8.23}$$

Let us now define

$$Z = X - Y - 1$$

Then equation (8.23) becomes

$$\frac{1}{R_y} = \frac{Z}{R_F}$$

Z must be positive in order for R_y to be physically realizable. If Z is negative, Case I does not apply.

Equations (8.15) and (8.16) yield the resistance values

$$R_i = \frac{R_F}{X_i}; \quad R_j = \frac{R_F}{Y_j} \tag{8.24}$$

Case II When $R_y \to \infty$, equation (8.21) becomes

$$\frac{1}{R_x} + \frac{X}{R_F} = \frac{1}{R_F} + \frac{Y}{R_F} \tag{8.25}$$

and

$$\frac{1}{R_x} = \frac{-(X - Y - 1)}{R_F} = \frac{-Z}{R_F} \tag{8.26}$$

Hence, Z must be negative.

Case III When $R_x \to \infty$ and $R_y \to \infty$, equation (8.21) becomes

Table 8.1 Summary of Summing Amplifier Design

Z	R_y	R_x	R_i	R_j
>0	$\frac{R_F}{Z}$	∞		
<0	∞	$\frac{R_F}{-Z}$	$\frac{R_F}{X_i}$	$\frac{R_F}{Y_j}$
0	∞	∞		

$$\frac{X}{R_F} = \frac{1}{R_F} + \frac{Y}{R_F} \tag{8.27}$$

and

$$0 = \frac{X - Y - 1}{R_F} = \frac{Z}{R_F} \tag{8.28}$$

Therefore, $Z = 0$.

The results of all three cases are combined in Table 8.1. Note that $Z = X - Y - 1$, where

$$X = \sum_{i=1}^{n} X_i$$

$$Y = \sum_{j=a}^{m} Y_j$$

Example 8.1 **Op-Amp Summer (Design)**

Design an op-amp summer to yield the following input/output relationship:

$$v_o = 10v_1 + 6v_2 + 4v_3 - 5v_a - 2v_b$$

SOLUTION The values of X, Y, and Z are calculated as follows:

$$X = \sum_{i=1}^{3} X_i = 10 + 6 + 4 = 20$$

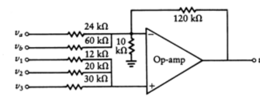

Figure 8.11 Amplifier for Example 8.1.

$$Y = \sum_{j=a}^{b} Y_j = 5 + 2 = 7$$

$$Z = X - Y - 1 = 20 - 7 - 1 = 12$$

In this example, Z is greater than zero, so we are dealing with Case I, where R_x is open-circuited. A suitable value of R_F must first be chosen. Once R_F is determined, all other resistor values are easily found. Suppose we want the minimum resistance, R_{min}, at any of the inputs to be 10 kΩ. Then the multiplying factor, K, would be the largest of any X_i, Y_j, or Z. Thus, $K = 12$ and $R_F = 10\ \text{k}\Omega \times 12 = 120\ \text{k}\Omega$. We do not want to choose too small a value, or the circuit will load previous circuitry. We also do not use an exceptionally large value for R_F, since this would increase the noise generated in that resistor. As a guide, all resistors used in the op-amp circuit should be between 1 kΩ and 1 MΩ. Having determined R_F, the resistors are found from equation (8.24) as follows:

$$R_1 = \frac{R_F}{X_1} = \frac{120\ \text{k}\Omega}{10} = 12\ \text{k}\Omega$$

$$R_2 = \frac{R_F}{X_2} = \frac{120\ \text{k}\Omega}{6} = 20\ \text{k}\Omega$$

$$R_3 = \frac{R_F}{X_3} = \frac{120\ \text{k}\Omega}{4} = 30\ \text{k}\Omega$$

$$R_a = \frac{R_F}{Y_a} = \frac{120\ \text{k}\Omega}{5} = 24\ \text{k}\Omega$$

$$R_b = \frac{R_F}{Y_b} = \frac{120\ \text{k}\Omega}{2} = 60\ \text{k}\Omega$$

$$R_y = \frac{R_F}{Z} = \frac{120\ \text{k}\Omega}{12} = 10\ \text{k}\Omega$$

The resulting amplifier is shown in Figure 8.11.

Example 8.2 **Op-Amp Summer (Design)**

Design an op-amp circuit to implement the following equation:

$$v_o = 4v_1 + v_2 - 8v_a - 6v_b$$

SOLUTION We first calculate the values of X, Y, and Z.

$$X = 4 + 1 = 5$$
$$Y = 8 + 6 = 14$$
$$Z = 5 - 14 - 1 = -10$$

Since Z is less than zero, R_y is open circuit and we are dealing with an example of Case II. Suppose in this case, we want the equivalent resistance at the + and − terminal to be 10 kΩ. Then the multiplying factor would be the largest of X or Y + 1. This would make $K = 15$ and $R_F = 15 \times 10\text{k} = 150\ \text{k}\Omega$.

$$R_x = \frac{R_F}{Z} = \frac{150\ \text{k}\Omega}{10} = 15\ \text{k}\Omega$$

$$R_a = \frac{R_F}{Y_a} = \frac{150\ \text{k}\Omega}{8} = 18.75\ \text{k}\Omega$$

$$R_b = \frac{R_F}{Y_b} = \frac{150\ \text{k}\Omega}{6} = 25\ \text{k}\Omega$$

$$R_1 = \frac{R_F}{X_1} = \frac{150\ \text{k}\Omega}{4} = 37.5\ \text{k}\Omega$$

$$R_2 = \frac{R_F}{X_2} = \frac{150\ \text{k}\Omega}{1} = 150\ \text{k}\Omega$$

The complete circuit is shown in Figure 8.12. Note that at each input terminal, the equivalent resistance is 10 kΩ or calculated as follows:

$$37.5\ \text{k}\Omega \parallel 150\ \text{k}\Omega \parallel 15\ \text{k}\Omega = 25\ \text{k}\Omega \parallel 18.75\ \text{k}\Omega \parallel 150\ \text{k}\Omega = 10\ \text{k}\Omega$$

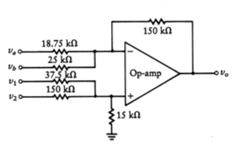

Figure 8.12 Amplifier for Example 8.2.

Recall that the derivation forced these to be equal to minimize the dc bias-current offset. Note also that if we are not happy with the resulting resistor values, they can all be multiplied by the same constant without changing the voltage relationships.

8.7 Other Op-Amp Applications

We have seen that the op-amp can be used as an amplifier, differentiator, or integrator and to combine a number of inputs in a linear manner. In this section, we investigate some additional important applications of this versatile IC.

8.7.1 *Negative Impedance Circuit*

The circuit shown in Figure 8.13 produces a negative input resistance (impedance in the general case). This can be used to cancel an unwanted positive resistance and thus produce an oscillator. The input resistance, R_{in}, is defined as

$$R_{in} = \frac{v}{i}$$

The op-amp inputs are given by

$$v_+ = v_- = v$$

As before, a voltage-divider relationship is used to derive the following expression:

$$v_- = v = \frac{R_A v_o}{R_A + R_F}$$

Solving for v_o in terms of v yields

$$v_o = v\left(1 + \frac{R_F}{R_A}\right)$$

Since the input impedance to the v_+ terminal is infinite, the current in R is equal to i and can be found as follows:

$$i = \frac{v - v_o}{R} = \frac{v - v(1 + R_F/R_A)}{R} = \frac{-R_F v}{R_A R}$$